THE DYNKIN DIAGRAMS PACKAGE
VERSION 3.14159265358979

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Contents

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1. Quick introduction

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of \(B_3\) is \dynkin B3.
\end{document}
```

Invoke it

```
The Dynkin diagram of \(B_3\) is \dynkin B3.
```

The Dynkin diagram of B_3 is .

Indefinite rank Dynkin diagrams

```
\dynkin B{}
```


Inside a TikZ statement

```
The Dynkin diagram of \(B_3\) is
\tikz \dynkin B3;
```

The Dynkin diagram of B_3 is

Inside a Dynkin diagram environment

```
The Dynkin diagram of \(B_3\) is
\begin{dynkinDiagram}B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{dynkinDiagram}
```

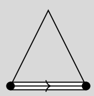
The Dynkin diagram of B_3 is

2. Interaction with TikZ

Inside a TikZ environment, default behaviour is to draw from the origin, so you can draw around the diagram:

Inside a TikZ environment

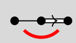
```
\begin{tikzpicture}
\draw (0,0) -- (.5,1) -- (1,0);
\dynkin[edge length=1cm]G2
\end{tikzpicture}
```



But it looks bad in the middle of text:

Inside a TikZ environment

```
The Dynkin diagram of \(B_3\) is
\begin{tikzpicture}[baseline]
\dynkin B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{tikzpicture}
```

The Dynkin diagram of B_3 is 

A vertical shift realigns the diagram to ambient text:

Inside a TikZ environment

```
The Dynkin diagram of \(B_3\) is
\begin{tikzpicture}[baseline]
\dynkin[vertical shift] B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{tikzpicture}
```

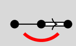
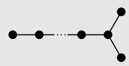
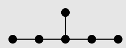
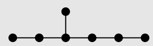
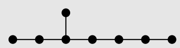
The Dynkin diagram of B_3 is 

Table 1: The Dynkin diagrams of the reduced simple root systems [3] pp. 265–290, plates I–IX

A_n		`\dynkin A{}`
C_n		`\dynkin C{}`
D_n		`\dynkin D{}`
E_6		`\dynkin E6`
E_7		`\dynkin E7`
E_8		`\dynkin E8`
F_4		`\dynkin F4`
G_2		`\dynkin G2`

3. Set options globally

Most options set globally . . .

```
\pgfkeys{/Dynkin diagram,
   edge length=.5cm,
   fold radius=.5cm,
   indefinite edge/.style={
      draw=black,
      fill=white,
      thin,
      densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger:* spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as a global option; moreover you should drop the extension `/.style` on any option with spaces in its name (but not otherwise). For example,

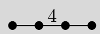
. . . or pass global options to the package

```
\usepackage[
    ordering=Kac,
    edge/.style=blue,
    indefinite-edge={draw=green,fill=white,densely dashed},
    indefinite-edge-ratio=5,
    mark=o,
    root-radius=.06cm]
    {dynkin-diagrams}
```

4. Coxeter diagrams

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```

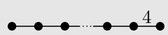
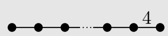
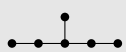
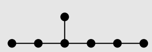
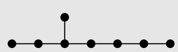
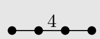
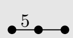
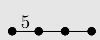


gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]G2\), \
\(I_n=\dynkin[Coxeter,gonality=n]I{}\)
```

$G_2 =$ , $I_n =$ 

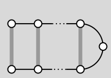
Table 2: The Coxeter diagrams of the simple reflection groups

A_n		`\dynkin[Coxeter]A{}`
B_n		`\dynkin[Coxeter]B{}`
C_n		`\dynkin[Coxeter]C{}`
E_6		`\dynkin[Coxeter]E6`
E_7		`\dynkin[Coxeter]E7`
E_8		`\dynkin[Coxeter]E8`
F_4		`\dynkin[Coxeter]F4`
G_2		`\dynkin[Coxeter,gonality=n]G2`
H_3		`\dynkin[Coxeter]H3`
H_4		`\dynkin[Coxeter]H4`
I_n		`\dynkin[Coxeter,gonality=n]I{}`

5. Satake diagrams

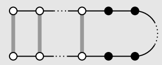
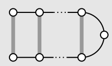
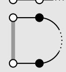
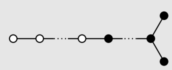
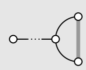
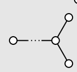
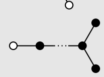
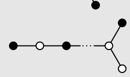
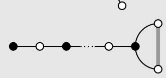
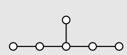
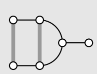
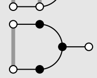
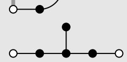
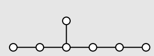
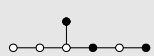
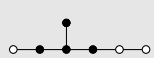
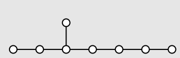
Satake diagrams use the standard name instead of a rank

```
\(A_{IIIb}=\dynkin A{IIIb}\)
```

$A_{IIIb} =$ 

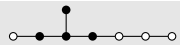
We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

A_I		\dynkin AI
A_{II}		\dynkin A{II}
A_{IIIa}		\dynkin A{IIIa}
A_{IIIb}		\dynkin A{IIIb}
A_{IV}		\dynkin A{IV}
B_I		\dynkin BI
B_{II}		\dynkin B{II}
C_I		\dynkin CI
C_{IIa}		\dynkin C{IIa}
C_{IIb}		\dynkin C{IIb}
D_{Ia}		\dynkin D{Ia}
D_{Ib}		\dynkin D{Ib}
D_{Ic}		\dynkin D{Ic}
D_{II}		\dynkin D{II}
D_{IIIa}		\dynkin D{IIIa}
D_{IIIb}		\dynkin D{IIIb}
E_I		\dynkin EI
E_{II}		\dynkin E{II}
E_{III}		\dynkin E{III}
E_{IV}		\dynkin E{IV}
E_V		\dynkin EV
E_{VI}		\dynkin E{VI}
E_{VII}		\dynkin E{VII}
E_{VIII}		\dynkin E{VIII}

continued . . .

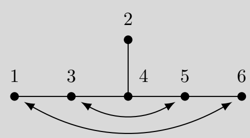
Table 3: . . . continued

E_{IX}		`\dynkin E{IX}`
F_I		`\dynkin FI`
F_{II}		`\dynkin F{II}`
G_I		`\dynkin GI`

6. How to fold

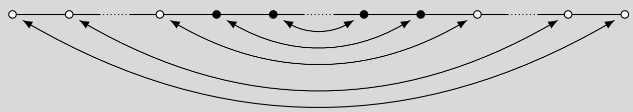
If you don't like the solid gray "folding bar", most people use arrows. Here is E_{II}

```
\dynkin[%
  edge length=.75cm,
  labels*={1,...,6},
  involutions={16;35}]E6
```



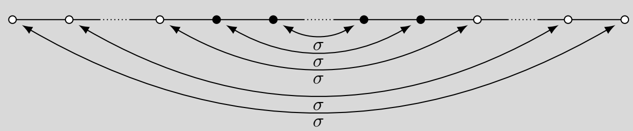
The double arrows for A_{IIIa} are big

```
\dynkin[edge length=.75cm,
   involutions={1{10};29;38;47;56}]{A}{oo.o**.**o.oo}
```



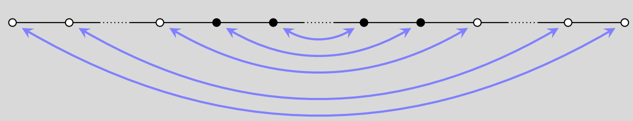
We can add labels

```
\dynkin[edge length=.75cm,
   involutions={
      1<below>[\sigma]{10};
      2<below>[\sigma]9;
      3<below>[\sigma]8;
      4<below>[\sigma]7;
      5<below>[\sigma]6}
]{A}{oo.o**.**o.oo}
```



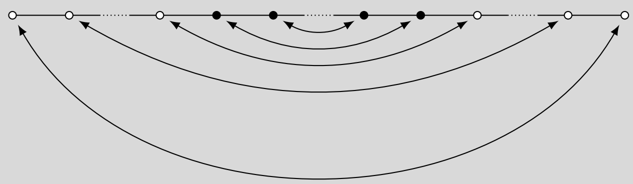
Style options

```
\dynkin[%
   edge length=.75cm,
      involution/.style={blue!50,stealth-stealth,thick},
   involutions={1{10};29;38;47;56}
   ]{A}{oo.o**.**o.oo}
```



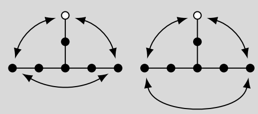
Arrow angles

```
\dynkin[%
    edge length=.75cm,
    involutions={[in=-120,out=-60,relative]1{10};29;38;47;56}
    ]{A}{oo.o**.**o.oo}
```



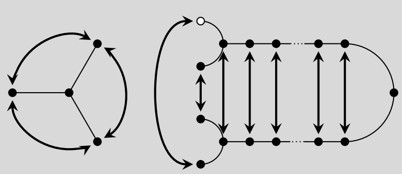
Arrow angles

```
\dynkin[involutions={16;60;01}]E[1]{6}
\dynkin[involutions={[out=-80,in=-100,relative]16;60;01}]E[1]{6}
```



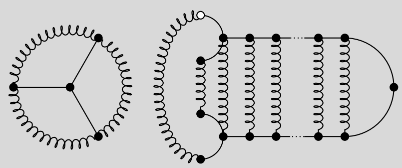
If you don’t like the solid gray “folding bar”, most people use arrows . . .

```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
        \dynkinFold 1{13}
        \dynkinFold[bend right=90] 0{14}
\end{dynkinDiagram}
```



. . . but you could try springs pulling roots together

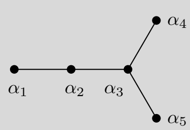
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
        \dynkinFold 1{13}
        \dynkinFold[bend right=90]0{14}
\end{dynkinDiagram}
```



7. Labels for the roots

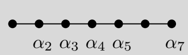
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},edge
      length=.75cm]D5
```



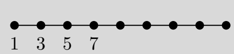
Labelling several roots

```
\dynkin[labels={,2,...,5,,7},label macro/.code={\alpha_{\drlap#1}}]A7
```



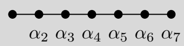
The **foreach** notation I

```
\dynkin[labels={1,3,...,7}]A9
```



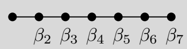
The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_...,\alpha_7}]A7
```



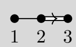
The foreach notation III

```
\dynkin[label macro/.code={\beta_{\drlap{#1}}},labels={,2,...,7}]A7
```



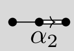
Label the roots individually by root number

```
\dynkin[label]B3
```



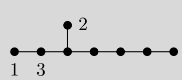
Access root labels via TikZ

```
\begin{dynkinDiagram}B3
\node[below] at (root 2) {\(\alpha_{\drlap{2}}\)};
\end{dynkinDiagram}
```



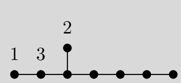
The labels have default locations, mostly below roots

```
\dynkin[labels={1,2,3}]E8
```



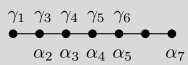
The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[labels*={1,2,3}]E8
```



Labelling several roots and alternates

```
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
```

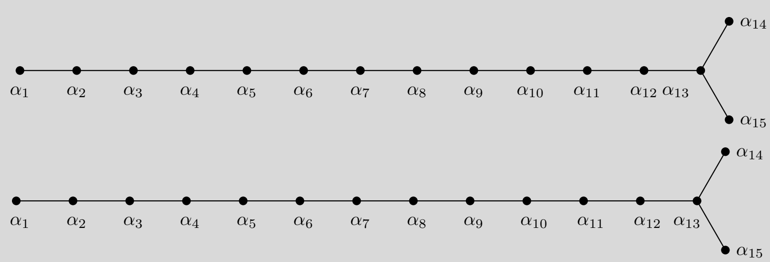


8. Label subscripts

Note the slight improvement that `\drlap` makes: the labels are centered on the middle of the letter α, ignoring the space taken up by the subscripts, using the **mathtools** command `\mathrlap`, but only for labels which are *not* placed to the left or right of a root.

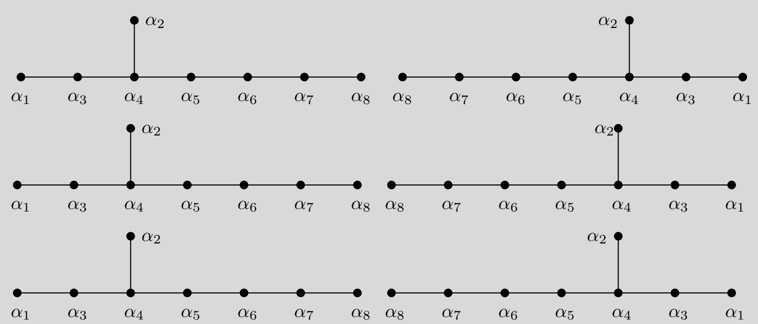
Label subscript spacing

```
\dynkin[label,label macro/.code={\alpha_{#1}},
    edge length=.75cm]D{15}
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
    edge length=.75cm]D{15}
```



Label subscript spacing

```
\dynkin[label,label macro/.code={\alpha_{#1}},
      edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{#1}},backwards,
      edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},
      edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},backwards,
      edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
      edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},backwards,
      edge length=.75cm]E8
```

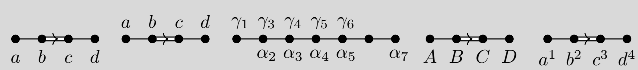


9. Height and depth of labels

Labels are set with default maximum height the height of the character b, and default maximum depth the depth of the character g. To change these, set `label height` and `label depth`:

Change height and depth of characters

```
\dynkin[labels={a,b,c,d},label height=d,label depth=d]F4
\dynkin[labels*={a,b,c,d},label height=d,label depth=d]F4
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
label height=$\alpha_1$,
label depth=$\alpha_1$,
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
\dynkin[labels={A,B,C,D},label height=$A$,label depth=$A$]F4
\dynkin[labels={a^1,b^2,c^3,d^4},label height=$X^X$]F4
```



10. Text style for the labels

Use a text style: big and blue

```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\alpha_{\drlap{#1}}}
]A{}
\end{dynkinDiagram}
```

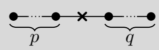
Use a text style; font selection is in the label macro

```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\mathbb{A}_{\drlap{#1}}}]A{}
\end{dynkinDiagram}
```

11. Bracing roots

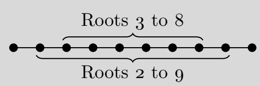
Bracing roots

```
\begin{dynkinDiagram}A{*.*x*.*}
\dynkinBrace[p]12
\dynkinBrace[q]45
\end{dynkinDiagram}
```



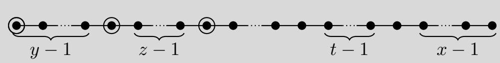
Bracing roots, and a starred form

```
\begin{dynkinDiagram}A{10}
\dynkinBrace[\text{Roots 2 to 9}]29
\dynkinBrace*[\text{Roots 3 to 8}]38
\end{dynkinDiagram}
```



Bracing roots

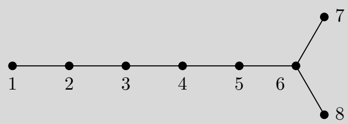
```
\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}A{**.***.***.***.***.**}
\circleRoot 4\circleRoot 7\circleRoot 10\circleRoot 13
\dynkinBrace[y-1]13
\dynkinBrace[z-1]56
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}
```



12. Label placement

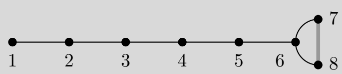
Take a D_8:

```
\dynkin[label,edge length=.75cm]D8
```



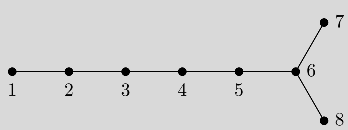
If you want to fold this diagram,

```
\dynkin[fold right,label,edge length=.75cm]D8
```



you will be glad that the 6 sits where it does, under and to the left. If you don't want to fold, you might prefer instead to put the 6 on the right side.

```
\dynkin[label,edge length=.75cm,label directions={,,,,,right,,}]D8
```



The default locations are overridden by the `label directions`. For extended diagrams, this list starts at 0-offset.

```
\dynkin[%
  label,
  label directions={above,,,,,,},
  involutions={[out=-60,in=-120,relative]16;60;01}
  ]E[1]{6}
```

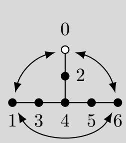
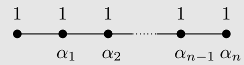
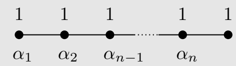
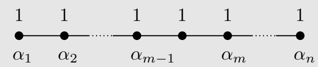
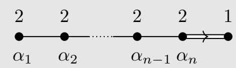
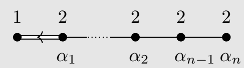
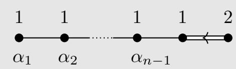
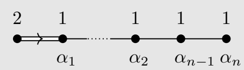
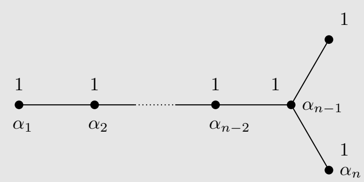
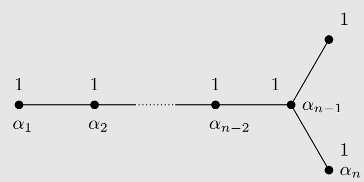
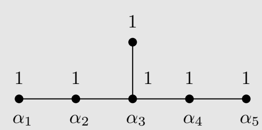
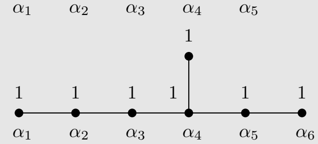
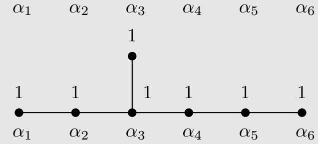
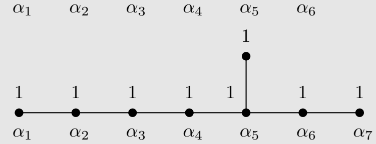


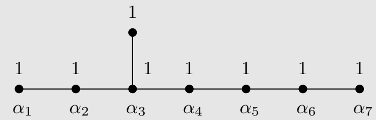
Table 4: Dynkin diagrams from Euler products [17]

i.	
ii.	
iii.	
iv.	
v.	
vi.	
vii.	
viii.	
ix.	
x.	
xi.	
xii.	
xiii.	

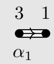
continued . . .

Table 4: …continued

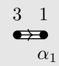
xiv.



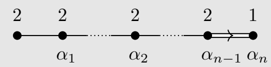
xv.



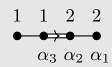
xvi.



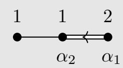
xvii.



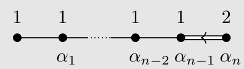
xviii.



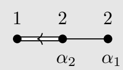
xix.



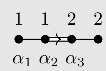
xx.



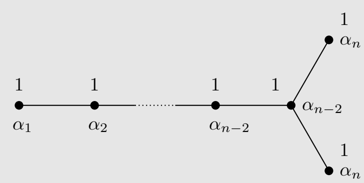
xxi.



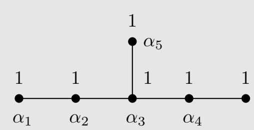
xxii.



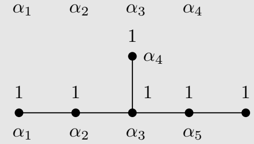
xxiii.



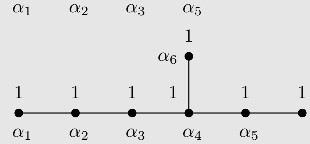
xxiv.



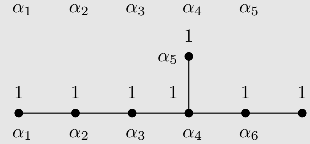
xxv.



xxvi.

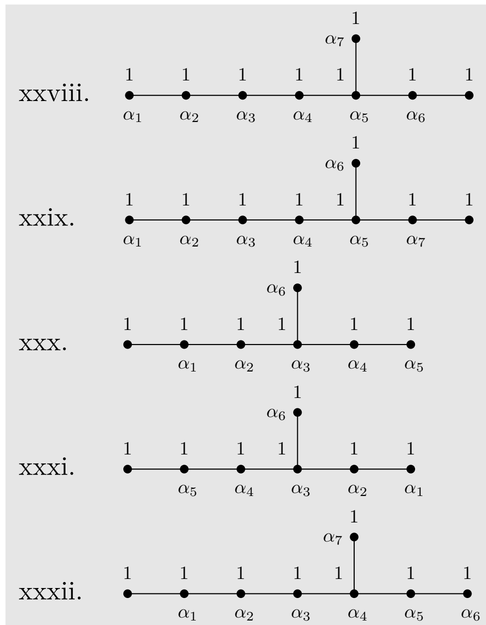


xxvii.



continued …

Table 4: ... continued



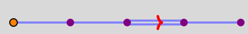

```
\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{\drlap{#1}}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
   \stepcounter{EPNo}\roman{EPNo}. &%
   \def\eL{.6cm}%
   \IfStrEqCase{#2}%
   {%
      D{%
         \gdef\eL{1cm}%
         \tikzset{/Dynkin diagram/label directions={,,,right,,}}%
      }%
      E{\gdef\eL{.75cm}}%
      F{\gdef\eL{.35cm}}%
      G{\gdef\eL{.35cm}}%
   }%
   \IfBooleanTF{#1}%
   {%
      \dynkin[edge length=\eL,backwards,labels*={#4},labels={#5}]{#2}{#3}
   }%
   {%
      \dynkin[edge length=\eL,labels*={#4},labels={#5}]{#2}{#3}
   }%
   \tikzset{/Dynkin diagram/label directions={}}%
   \\%
}%
\renewcommand*\do[1]{\EP#1}%
```

```
\begin{longtable}{MM}
   \caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
   \endfirsthead
   \caption{\dots continued}\\
   \endhead
   \multicolumn{2}{c}{continued \dots}\\
   \endfoot
   \endlastfoot
   \docsvlist{
      A{***.**}{1,1,1,1,1}{,1,2,n-1,n},
      A{***.**}{1,1,1,1,1}{1,2,n-1,n},
      A{**.***.*}{1,1,1,1,1,1}{1,2,m-1,,m,n},
      B{**.***}{2,2,2,2,1}{1,2,n-1,n},
      *B{***.**}{2,2,2,2,1}{n,n-1,2,1,},
      C{**.***}{1,1,1,1,2}{1,2,n-1,},
      *C{***.**}{1,1,1,1,2}{n,n-1,2,1,},
      D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
      D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
      E6{1,1,1,1,1,1}{1,...,5},
      *E7{1,1,1,1,1,1,1}{6,...,1},
      E7{1,1,1,1,1,1,1}{1,...,6},
      *E8{1,1,1,1,1,1,1,1}{7,...,1},
      E8{1,1,1,1,1,1,1,1}{1,...,7},
      G2{1,3}{,1},
      G2{1,3}{1},
      B{**.*.**}{2,2,2,2,1}{,1,2,n-1,n},
      F4{1,1,2,2}{,3,2,1},
      C3{1,1,2}{,2,1},
      C{**.***}{1,1,1,1,2}{,1,n-2,n-1,n},
      *B3{2,2,1}{1,2},
      F4{1,1,2,2}{1,2,3},
      D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n},
      E6{1,1,1,1,1,1}{1,2,3,4,,5},
      E6{1,1,1,1,1,1}{1,2,3,5,,4},
      *E7{1,1,1,1,1,1,1}{,5,...,1,6},
      *E7{1,1,1,1,1,1,1}{,6,4,3,2,1,5},
      *E8{1,1,1,1,1,1,1,1}{,6,...,1,7},
      *E8{1,1,1,1,1,1,1,1}{,7,5,4,3,2,1,6},
      *E7{1,1,1,1,1,1,1}{5,...,1,,6},
      *E7{1,1,1,1,1,1,1}{1,...,5,,6},
      *E8{1,1,1,1,1,1,1,1}{6,...,1,,7}%
      }
\end{longtable}
```

13. Style

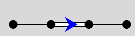
Colours

```
\dynkin[extended,
        o/.append style={fill=orange},
        */.style=blue!50!red,
        edge length=.75cm,
        edge/.style={blue!50,thick},
        arrow width=2mm,
        arrow style={red,width=2mm,line width=1pt}]{F}{4}
```



Arrow shapes

```
\dynkin[edge length=.5cm,
    arrow width=2mm,
    arrow shape/.style={-{Stealth[blue,width=2mm]}}]F4
```



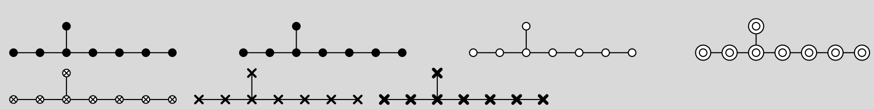
Edge lengths

```
The Dynkin diagram of \(A_3\) is \dynkin[edge length=1.2]A3
```

The Dynkin diagram of A_3 is

Root marks

```
\dynkin E8
\dynkin[mark=*]E8
\dynkin[mark=o]E8
\dynkin[mark=O]E8
\dynkin[mark=t]E8
\dynkin[mark=x]E8
\dynkin[mark=X]E8
```

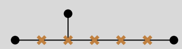


At the moment, you can only use:

`*`	•	solid dot
`o`	○	hollow circle
`O`	◎	double hollow circle
`t`	⊗	tensor root
`x`	×	crossed root
`X`	✖	thickly crossed root

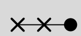
Mark styles

```
The parabolic subgroup \(E_{8,124}\) is
    \dynkin[parabolic=124,x/.style={brown,very thick}]E8
```

The parabolic subgroup $E_{8,124}$ is 

Sizes of root marks

```
\(A_{3,3}\) with big root marks is \dynkin[root
    radius=.08cm,parabolic=3]A3
```

$A_{3,3}$ with big root marks is 

14. Suppress or reverse arrows

Some diagrams have double or triple edges

```
\dynkin F4
\dynkin G2
```


Suppress arrows

```
\dynkin[arrows=false]F4
\dynkin[arrows=false]G2
```

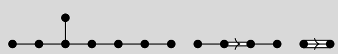
Reverse arrows

```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```


15. Backwards and upside down

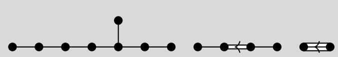
Default

```
\dynkin E8
\dynkin F4
\dynkin G2
```



Backwards

```
\dynkin[backwards]E8
\dynkin[backwards]F4
\dynkin[backwards]G2
```



Reverse arrows

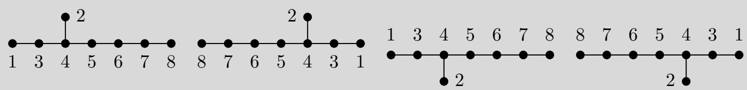
```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```


Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]F4
\dynkin[backwards,reverse arrows]G2
```

Backwards versus upside down

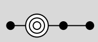
```
\dynkin[label]E8
\dynkin[label,backwards]E8
\dynkin[label,upside down]E8
\dynkin[label,backwards,upside down]E8
```



16. Drawing on top of a Dynkin diagram

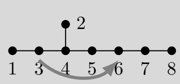
TikZ can access the roots themselves

```
\begin{dynkinDiagram}A4
    \fill[white,draw=black] (root 2) circle (.15cm);
    \fill[white,draw=black] (root 2) circle (.1cm);
    \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



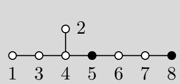
Draw curves between the roots

```
\begin{dynkinDiagram}[label]E8
    \draw[very thick, black!50,-latex]
        (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



Change marks

```
\begin{dynkinDiagram}[mark=o,label]E8
    \dynkinRootMark{*}5
    \dynkinRootMark{*}8
\end{dynkinDiagram}
```

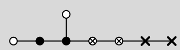


17. Mark lists

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin E{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, . . . , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin A{x4o3t4}
```


Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

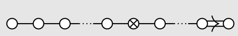
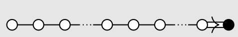
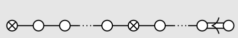
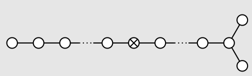
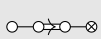
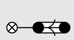
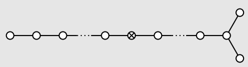
		\tikzset{/Dynkin diagram,root radius=.07cm}
A_{mn}		\dynkin A{o3.oto.oo}
B_{mn}		\dynkin B{o3.oto.oo}
B_{0n}		\dynkin B{o3.o3.o*}
C_n		\dynkin C{too.oto.oo}
D_{mn}		\dynkin D{o3.oto.o4}
$D_{21\alpha}$		\dynkin A{oto}
F_4		\dynkin F{ooot}
G_3		\dynkin[extended,affine mark=t, reverse arrows]G2

Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

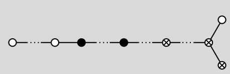
A_{mn}		`\dynkin A{o3.oto.oo}`
B_{mn}		`\dynkin B{o3.oto.oo}`
B_{0n}		`\dynkin B{o3.o3.o*}`
C_n		`\dynkin C{too.oto.oo}`
D_{mn}		`\dynkin D{o3.oto.o4}`
$D_{21\alpha}$		`\dynkin A{oto}`
F_4		`\dynkin F{ooot}`
G_3		`\dynkin[extended,affine mark=t, reverse arrows]G2`

18. Indefinite edges

An *indefinite edge* is a dashed edge between two roots, indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

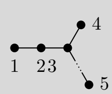
```
\dynkin D{o.o*.*.t.to.t}
```



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

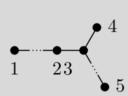
Indefinite edge option

```
\dynkin[make indefinite edge={3-5},label]D5
```



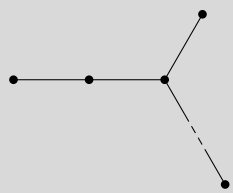
Give a list of edges to become indefinite

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]D5
```



Indefinite edge style

```
\dynkin[indefinite edge/.style={
        draw=black,fill=white,thin,densely dashed},
        edge length=1cm,
        make indefinite edge={3-5}]D5
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,
        indefinite edge ratio=3,
        make indefinite edge={3-5}]D5
```

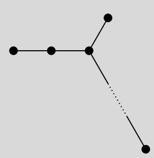
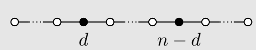
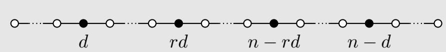
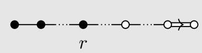
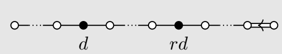
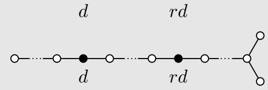
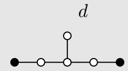
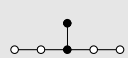
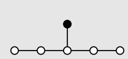
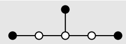
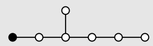
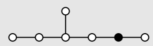
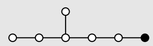
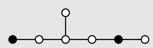
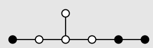
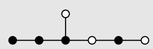
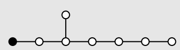
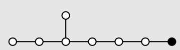
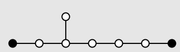
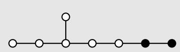
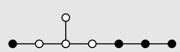
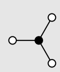


Table 7: Springer's table of indices [24], pp. 320-321, with one form of E_7 corrected

A_n		
A_n		
B_n		
C_n		
D_n		
E_6		`\dynkin E{*oooo*}`
E_6		`\dynkin E{o*o*oo}`
E_6		`\dynkin E{o*oooo}`

continued ...

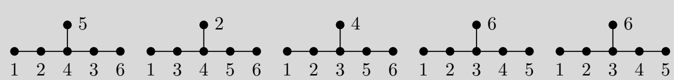
Table 7: . . . continued

E_6		`\dynkin E{**ooo*}`
E_7		`\dynkin E{*oooooo}`
E_7		`\dynkin E{ooooo*o}`
E_7		`\dynkin E{oooooo*}`
E_7		`\dynkin E{*oooo*o}`
E_7		`\dynkin E{*oooo**}`
E_7		`\dynkin E{*o**o*o}`
E_8		`\dynkin E{*ooooooo}`
E_8		`\dynkin E{ooooooo*}`
E_8		`\dynkin E{*oooooo*}`
E_8		`\dynkin E{oooooo**}`
E_8		`\dynkin E{*oooo***}`
F_4		`\dynkin F{ooo*}`
D_4		`\dynkin D{o*oo}`

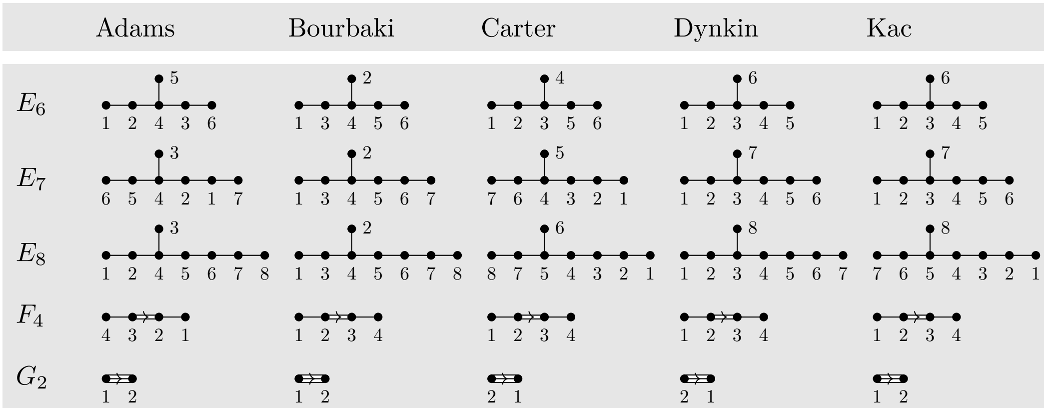
19. Root ordering

Root ordering

```
\dynkin[label,ordering=Adams]E6
\dynkin[label,ordering=Bourbaki]E6
\dynkin[label,ordering=Carter]E6
\dynkin[label,ordering=Dynkin]E6
\dynkin[label,ordering=Kac]E6
```

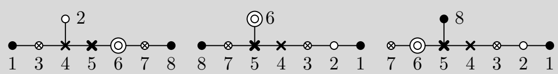


Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I-IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.



The marks are set down in order according to the current root ordering:

```
\dynkin[label]E{*otxXOt*}
\dynkin[label,ordering=Carter]E{*otxXOt*}
\dynkin[label,ordering=Kac]E{*otxXOt*}
```



Convert between orderings

```
\newcount\r
\dynkinOrder E8.Carter::6->Bourbaki.{\r}
In \(E_8\), root 6 in Carter's ordering is root \the\r{} in Bourbaki's
    ordering.
```

In E_8, root 6 in Carter's ordering is root 2 in Bourbaki's ordering.

20. Parabolic subgroups

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

```
The flag variety of pointed lines in
projective 3-space is associated to
the Dynkin diagram \dynkin[parabolic=3]A3.
```

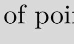
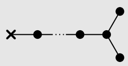
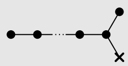
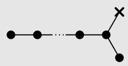
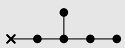
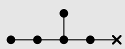
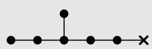
The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram .

Table 9: The Hermitian symmetric spaces

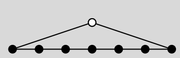
A_n		Grassmannian of k-planes in $\mathbb{C}^{n+1}$
B_n		$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$
C_n		space of Lagrangian n-planes in $\mathbb{C}^{2n}$
D_n		$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$
D_n		one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$
D_n		the other component
E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space

```
\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}l}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\\ \endhead
\caption{continued \dots}\\ \endfoot
\endlastfoot
\HSS{A_n}A{**.*x*.**}{Grassmannian of $k$-planes in $\C{n+1}$}
\HSS{B_n}[1]B{}{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n+1}$}
\HSS{C_n}[16]C{}{space of Lagrangian $n$-planes in $\C{2n}$}
\HSS{D_n}[1]D{}{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n}$}
\HSS{D_n}[32]D{}{one component of the variety of maximal dimension null subspaces of $\C{2n}$}
\HSS{D_n}[16]D{}{the other component}
\HSS{E_6}[1]E6{complexified octave projective plane}
\HSS{E_6}[32]E6{its dual plane}
\HSS{E_7}[64]E7{the space of null octave 3-planes in octave 6-space}
\end{longtable}
```

21. Extended Dynkin diagrams

Extended Dynkin diagrams

```
\dynkin[extended]A7
```



The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin A7` to become `\dynkin A[1]7`:

Extended Dynkin diagrams

```
\dynkin A[1]7
```

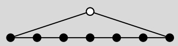
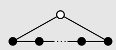
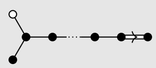
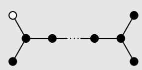
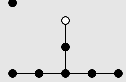
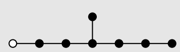
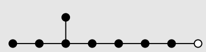
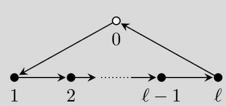


Table 10: The Dynkin diagrams of the extended simple root systems

A_1^1		`\dynkin[extended]A1`
A_n^1		`\dynkin[extended]A{}`
B_n^1		`\dynkin[extended]B{}`
C_n^1		`\dynkin[extended]C{}`
D_n^1		`\dynkin[extended]D{}`
E_6^1		`\dynkin[extended]E6`
E_7^1		`\dynkin[extended]E7`
E_8^1		`\dynkin[extended]E8`
F_4^1		`\dynkin[extended]F4`
G_2^1		`\dynkin[extended]G2`

Directed edges

```
\dynkin[%
   edge length=.75cm,
   edge/.style={-{stealth[sep=2pt]}},
   labels={,1,2,\ell-1,\ell},
   labels*={0}]
A[1]{}
```



22. Affine twisted and untwisted Dynkin diagrams

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

```
\(A^{(1)}_7=\dynkin A[1]7, \
E^{(2)}_6=\dynkin E[2]6, \
D^{(3)}_4=\dynkin D[3]4\)
```

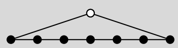
$A^{(1)}_7 =$ , $E^{(2)}_6 =$, $D^{(3)}_4 =$

Table 11: The affine Dynkin diagrams

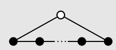
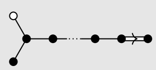
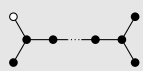
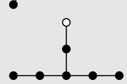
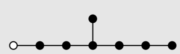
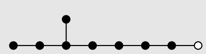
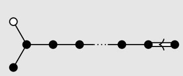
A^1_1		`\dynkin A[1]1`
A^1_n		`\dynkin A[1]{}`
B^1_n		`\dynkin B[1]{}`
C^1_n		`\dynkin C[1]{}`
D^1_n		`\dynkin D[1]{}`
E^1_6		`\dynkin E[1]6`
E^1_7		`\dynkin E[1]7`
E^1_8		`\dynkin E[1]8`
F^1_4		`\dynkin F[1]4`
G^1_2		`\dynkin G[1]2`
A^2_2		`\dynkin A[2]2`
A^2_{ev}		`\dynkin A[2]{even}`
A^2_{od}		`\dynkin A[2]{odd}`
D^2_n		`\dynkin D[2]{}`
E^2_6		`\dynkin E[2]6`
D^3_4		`\dynkin D[3]4`

Table 12: Some more affine Dynkin diagrams

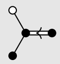
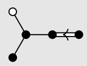
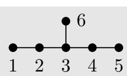
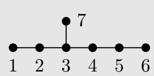
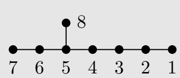
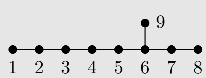
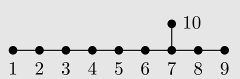
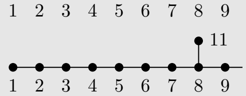
A_4^2		\dynkin A[2]4
A_5^2		\dynkin A[2]5
A_6^2		\dynkin A[2]6
A_7^2		\dynkin A[2]7
A_8^2		\dynkin A[2]8
D_3^2		\dynkin D[2]3
D_4^2		\dynkin D[2]4
D_5^2		\dynkin D[2]5
D_6^2		\dynkin D[2]6
D_7^2		\dynkin D[2]7
D_8^2		\dynkin D[2]8
D_4^3		\dynkin D[3]4
E_6^2		\dynkin E[2]6

Table 13: Some more Kac–Moody Dynkin diagrams, only allowed in Kac ordering

E_6		\dynkin[ordering=Kac,label]E6
E_7		\dynkin[ordering=Kac,label]E7
E_8		\dynkin[ordering=Kac,label]E8
E_9		\dynkin[ordering=Kac,label]E9
E_{10}		\dynkin[ordering=Kac,label]E{10}
E_{11}		\dynkin[ordering=Kac,label]E{11}

23. Extended Coxeter diagrams

Extended and Coxeter options together

```
\dynkin[extended,Coxeter]F4
```

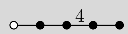
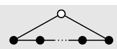
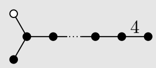
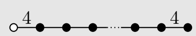
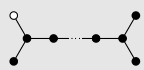
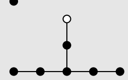
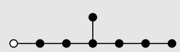
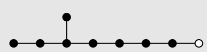
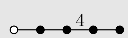
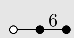
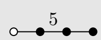
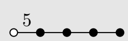


Table 14: The extended (affine) Coxeter diagrams

A_n		\dynkin[extended,Coxeter]A{}
B_n		\dynkin[extended,Coxeter]B{}
C_n		\dynkin[extended,Coxeter]C{}
D_n		\dynkin[extended,Coxeter]D{}
E_6		\dynkin[extended,Coxeter]E6
E_7		\dynkin[extended,Coxeter]E7
E_8		\dynkin[extended,Coxeter]E8
F_4		\dynkin[extended,Coxeter]F4
G_2		\dynkin[extended,Coxeter]G2
H_3		\dynkin[extended,Coxeter]H3
H_4		\dynkin[extended,Coxeter]H4
I_1		\dynkin[extended,Coxeter]I1

24. Kac style

We include a style called `Kac` which tries to imitate the style of [15].

Kac style

```
\dynkin[Kac]F4
```

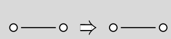


Table 15: The Dynkin diagrams of the simple root systems in Kac style

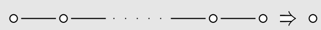
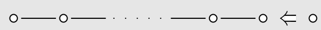
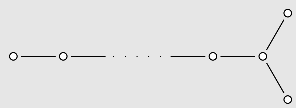
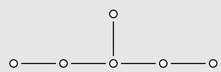
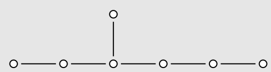
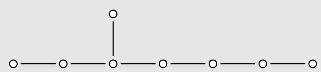
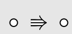
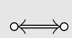
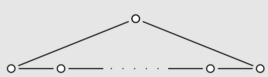
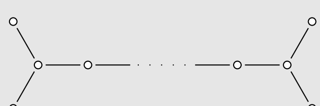
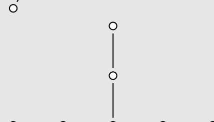
A_n		`\dynkin A{}`
B_n		`\dynkin B{}`
C_n		`\dynkin C{}`
D_n		`\dynkin D{}`
E_6		`\dynkin E6`
E_7		`\dynkin E7`
E_8		`\dynkin E8`
F_4		`\dynkin F4`
G_2		`\dynkin G2`

Table 16: The Dynkin diagrams of the extended simple root systems in Kac style

A_1^1		`\dynkin[extended]A1`
A_n^1		`\dynkin[extended]A{}`
B_n^1		`\dynkin[extended]B{}`
C_n^1		`\dynkin[extended]C{}`
D_n^1		`\dynkin[extended]D{}`
E_6^1		`\dynkin[extended]E6`
E_7^1		`\dynkin[extended]E7`
E_8^1		`\dynkin[extended]E8`

continued . . .

Table 16: . . . continued

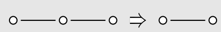
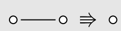
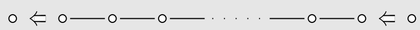
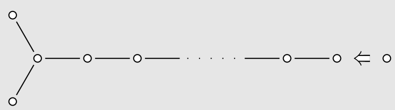
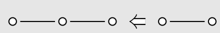
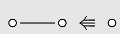
F_4^1		\dynkin[extended]F4
G_2^1		\dynkin[extended]G2

Table 17: The Dynkin diagrams of the twisted simple root systems in Kac style

A_2^2		\dynkin[extended]A[2]2
A_{ev}^2		\dynkin[extended]A[2]{even}
A_{od}^2		\dynkin[extended]A[2]{odd}
D_n^2		\dynkin[extended]D[2]{}
E_6^2		\dynkin[extended]E[2]6
D_4^3		\dynkin[extended]D[3]4

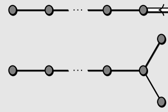
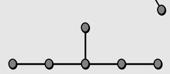
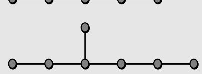
25. Ceref style

We include a style called `ceref` which paints oblong root markers with shadows. The word “ceref” is an old form of the word “serif”.

Ceref style

```
\dynkin[ceref]F4
```


Table 18: The Dynkin diagrams of the simple root systems in ceref style

A_n		\dynkin A{}
B_n		\dynkin B{}
C_n		\dynkin C{}
D_n		\dynkin D{}
E_6		\dynkin E6
E_7		\dynkin E7

continued . . .

Table 18: . . . continued

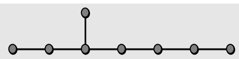
E_8		`\dynkin E8`
F_4		`\dynkin F4`
G_2		`\dynkin G2`

Table 19: The Dynkin diagrams of the extended simple root systems in ceref style

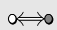
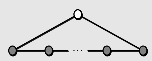
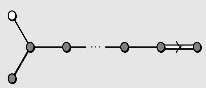
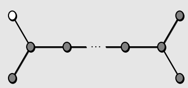
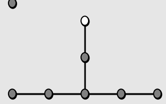
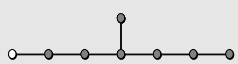
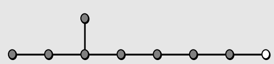
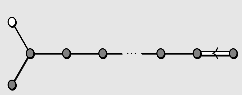
A_1^1		`\dynkin[extended]A1`
A_n^1		`\dynkin[extended]A{}`
B_n^1		`\dynkin[extended]B{}`
C_n^1		`\dynkin[extended]C{}`
D_n^1		`\dynkin[extended]D{}`
E_6^1		`\dynkin[extended]E6`
E_7^1		`\dynkin[extended]E7`
E_8^1		`\dynkin[extended]E8`
F_4^1		`\dynkin[extended]F4`
G_2^1		`\dynkin[extended]G2`

Table 20: The Dynkin diagrams of the twisted simple root systems in ceref style

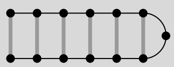
A_2^2		`\dynkin[extended]A[2]2`
A_{ev}^2		`\dynkin[extended]A[2]{even}`
A_{od}^2		`\dynkin[extended]A[2]{odd}`
D_n^2		`\dynkin[extended]D[2]{}`
E_6^2		`\dynkin[extended]E[2]6`
D_4^3		`\dynkin[extended]D[3]4`

26. More on folded Dynkin diagrams

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

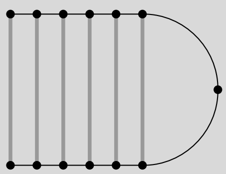
Folding

```
\dynkin[fold]A{13}
```



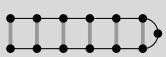
Big fold radius

```
\dynkin[fold,fold radius=1cm]A{13}
```



Small fold radius

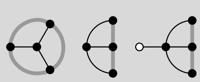
```
\dynkin[fold,fold radius=.2cm]A{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

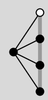
3-ply

```
\dynkin[ply=3]D4
\dynkin[ply=3,fold right]D4
\dynkin[ply=3]D[1]4
```



4-ply

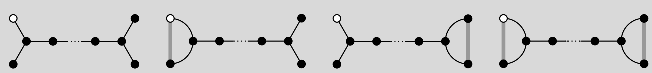
```
\dynkin[ply=4]D[1]4
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin D[1]{} \
\dynkin[fold left]D[1]{} \
\dynkin[fold right]D[1]{} \
\dynkin[fold]D[1]{}
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
	\dynkin[ply=4]D[1]{****.*****.*****}%
 \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
	\dynkinFold[bend right=90]1{13}%
	\dynkinFold[bend right=90]0{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
	\dynkinFold01%
	\dynkinFold1{13}%
	\dynkinFold{13}{14}%
\end{dynkinDiagram}
```

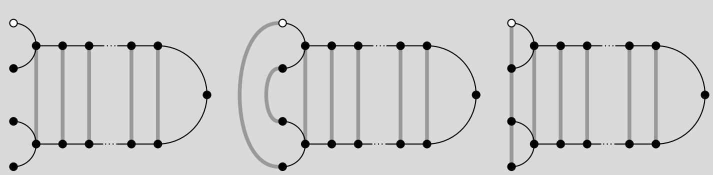
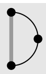
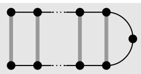
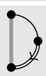
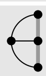
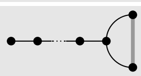
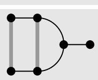
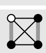
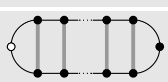
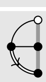
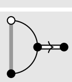
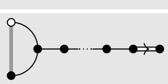
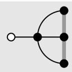
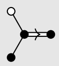
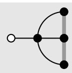
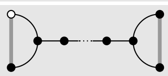
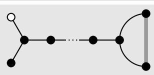
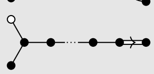
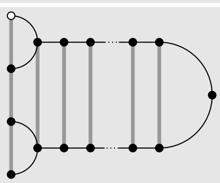
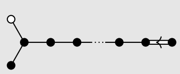
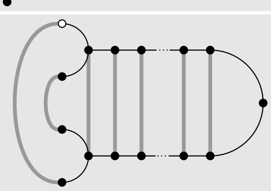
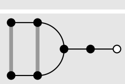
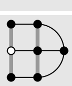


Table 21: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

A_3		`\dynkin[fold]A[0]3`
C_2		`\dynkin C[0]2`
$A_{2\ell-1}$		`\dynkin[fold]A{**.*****.**}`
C_ℓ		`\dynkin C{}`
B_3		`\dynkin[fold]B[0]3`
G_2		`\dynkin[reverse arrows]G[0]2`
D_4		`\dynkin[ply=3,fold right]D4`
G_2		`\dynkin G2`
$D_{\ell+1}$		`\dynkin[fold]D{}`
B_ℓ		`\dynkin B{}`
E_6		`\dynkin[fold]E[0]6`
F_4		`\dynkin[reverse arrows]F[0]4`
A_3^1		`\dynkin[ply=4]A[1]3`
A_1^1		`\dynkin A[1]1`
$A_{2\ell-1}^1$		`\dynkin[fold]A[1]{**.*****.**}`
C_ℓ^1		`\dynkin C[1]{}`
B_3^1		`\dynkin[ply=3]B[1]3`
A_2^2		`\dynkin A[2]2`
B_3^1		`\dynkin[ply=2]B[1]3`
G_2^1		`\dynkin G[1]2`
B_ℓ^1		`\dynkin[fold]B[1]{}`
D_ℓ^2		`\dynkin D[2]{}`

continued . . .

Table 21: . . . continued

D_4^1		\dynkin[ply=3]D[1]4
B_3^1		\dynkin B[1]3
D_4^1		\dynkin[ply=3]D[1]4
G_2^1		\dynkin G[1]2
$D_{\ell+1}^1$		\dynkin[fold]D[1]{}
D_ℓ^2		\dynkin D[2]{}
$D_{\ell+1}^1$		\dynkin[fold right]D[1]{}
B_ℓ^1		\dynkin B[1]{}
$D_{2\ell}^1$		\begin{dynkinDiagram}[ply=4]D[1]% {****.*****.*****} \dynkinFold01 \dynkinFold1{13} \dynkinFold{13}{14} \end{dynkinDiagram}
A_{odd}^2		\dynkin A[2]{odd}
$D_{2\ell}^1$		\begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold[bend right=90]1{13} \dynkinFold[bend right=90]0{14} \end{dynkinDiagram}
A_{even}^2		\dynkin A[2]{even}
E_6^1		\dynkin[fold]E[1]6
F_4^1		\dynkin[reverse arrows]F[1]4
E_6^1		\dynkin[ply=3]E[1]6
D_4^3		\dynkin D[3]4

continued . . .

Table 21: …continued

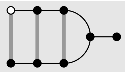
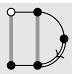
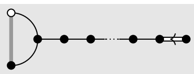
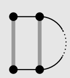
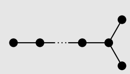
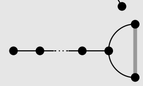
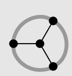
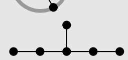
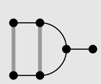
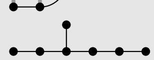
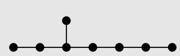
E_7^1		\dynkin[fold]E[1]7
E_6^2		\dynkin E[2]6
F_4^1		\dynkin[fold]F[1]4
G_2^1		\dynkin G[1]2
A^2_{odd}		\dynkin[odd,fold]A[2]{****.***}
A^2_{even}		\dynkin A[2]{even}
D_3^2		\dynkin[fold]D[2]3
A_2^2		\dynkin A[2]2

Table 22: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell\geq 1}$		\dynkin A{}
${}^2A_{\ell\geq 2}$		\dynkin[fold]A{}
$B_{\ell\geq 2}$		\dynkin B{}
${}^2B_2$		\dynkin[fold]B2
$C_{\ell\geq 3}$		\dynkin C{}
$D_{\ell\geq 4}$		\dynkin D{}
${}^2D_{\ell\geq 4}$		\dynkin[fold]D{}
${}^3D_4$		\dynkin[ply=3]D4
E_6		\dynkin E6
${}^2E_6$		\dynkin[fold]E6
E_7		\dynkin E7
E_8		\dynkin E8
F_4		\dynkin F4
${}^2F_4$		\dynkin[fold]F4

continued …

Table 22: ...continued

G_2		\dynkin G2
${}^2G_2$		\dynkin[fold]G2

27. Typesetting mathematical names of Dynkin diagrams

The `\dynkinName` command, with the same syntax as `\dynkin`, typesets a default name of your diagram in LaTeX. It is perhaps only useful when automatically generating a large collection of Dynkin diagrams in a computer program.

Name of a diagram

```
\dynkinName[label,extended]B7
\dynkinName A[2]{even}
\dynkinName[Coxeter]B7
\dynkinName[label,extended]B{}
\dynkinName D[3]4
```

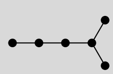
B^1_7 A^2_{ev} B_7 B^1_n D^3_4

28. Connecting Dynkin diagrams

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]D6
```

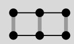


We can then connect the two with folding edges:

Connect diagrams

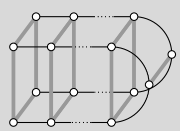
```
\begin{dynkinDiagram}[name=upper]A3
	\node (current) at ($(upper root 1)+(0,-.3cm)$) {};
	\dynkin[at=(current),name=lower]A3
	\begin{pgfonlayer}{Dynkin behind}
		\foreach \i in {1,...,3}%
		{%
			\draw[/Dynkin diagram/fold style]
				($(upper root \i)$)
				-- ($(lower root \i)$);%
		}%
```

```
	\end{pgfonlayer}
\end{dynkinDiagram}
```



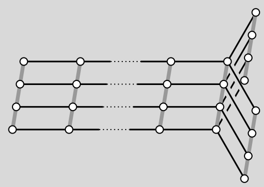
The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
	\dynkin[name=1]A{IIIb}
	\node (a) at (-.3,-.4){};
	\dynkin[name=2,at=(a)]A{IIIb}
	\begin{pgfonlayer}{Dynkin behind}
		\foreach \i in {1,...,7}%
		{%
			\draw[/Dynkin diagram/fold style]
				($(1 root \i)$)
				--
				($(2 root \i)$);%
		}%
	\end{pgfonlayer}
\end{tikzpicture}
```



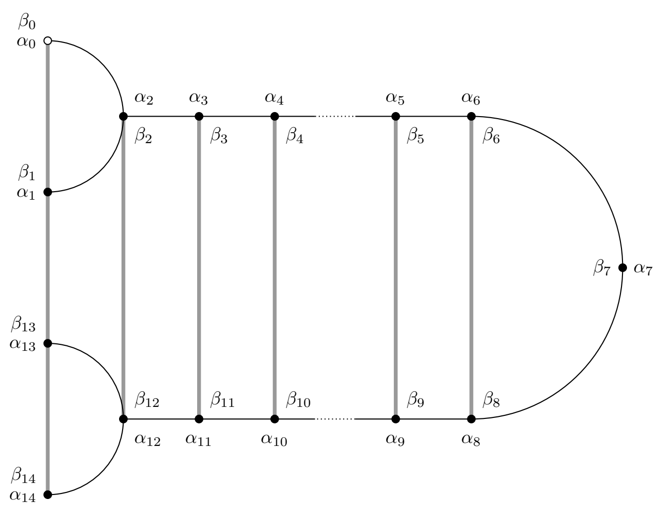
```
\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
	\foreach \d in {1,...,4}
	{
		\node (current) at ($(\d*.05,\d*.3)$){};
		\dynkin[name=\d,at=(current)]D{oo.oooo}
	}
	\begin{pgfonlayer}{Dynkin behind}
		\foreach \i in {1,...,6}%
		{%
			\draw[/Dynkin diagram/fold style] ($(1 root \i)$) -- ($(2
	root \i)$);%
			\draw[/Dynkin diagram/fold style] ($(2 root \i)$) -- ($(3
	root \i)$);%
```

```
            \draw[/Dynkin diagram/fold style] ($(3 root \i)$) -- ($(4
    root \i)$);%
          }%
      \end{pgfonlayer}
\end{tikzpicture}
```

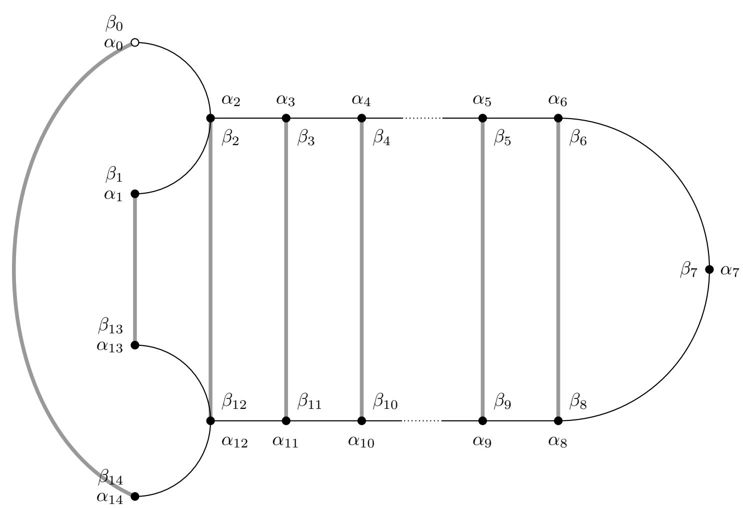


29. Other examples

$^1D_4$ 4-ply tied straight:



$^1D_4$ 4-ply tied bending:

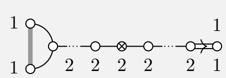


```
\tikzset{/Dynkin diagram,
    edge length=1cm,
    fold radius=1cm,
    label,
    label*=true,
    label macro/.code={\alpha_{#1}},
    label macro*/.code={\beta_{#1}}}
\({}^1 D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
    \dynkinFold 01
    \dynkinFold 1{13}
    \dynkinFold{13}{14}
\end{dynkinDiagram}
\({}^1 D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4,label]D[1]%
{****.*****.*****}
    \dynkinFold1{13}
    \dynkinFold[bend right=65]0{14}
\end{dynkinDiagram}
```

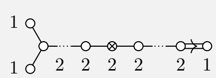
Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

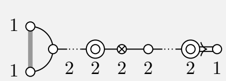
```
\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
      \dynkinLabelRoot*71
\end{dynkinDiagram}
```



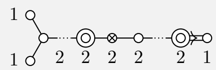

```
\dynkin[label]B[1]{oo.oto.oo}
```



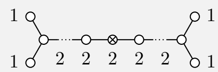

```
\dynkin[ply=2,label]B[1]{oo.Oto.Oo}
```



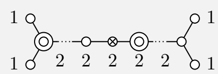
```
\dynkin[label]B[1]{oo.Oto.Oo}
```



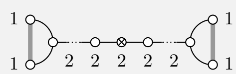

```
\dynkin[label]D[1]{oo.oto.ooo}
```




```
\dynkin[label]D[1]{oO.otO.ooo}
```

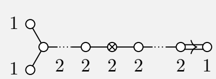



```
\dynkin[label,fold]D[1]{oo.oto.ooo}
```

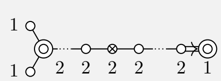


$\mathfrak{sl}(2m+1|2n)^2$

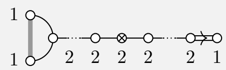
```
\dynkin[label]B[1]{oo.oto.oo}
```




```
\dynkin[label]B[1]{oO.oto.oO}
```

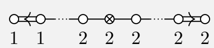


\dynkin[label,fold]B[1]{oo.oto.oo}

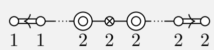


$\mathfrak{sl}\left(2m+1|2n+1\right)^2$

\dynkin[label]D[2]{o.oto.oo}

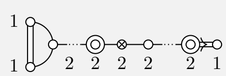


\dynkin[label]D[2]{o.OtO.oo}

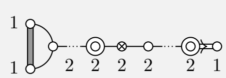


$\mathfrak{sl}\left(2|2n+1\right)^{(2)}$

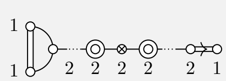
\dynkin[ply=2,label,double edges]B[1]{oo.Oto.Oo}



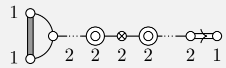
\dynkin[ply=2,label,double fold]B[1]{oo.Oto.Oo}



\dynkin[ply=2,label,double edges]B[1]{oo.OtO.oo}

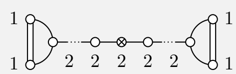


```
\dynkin[ply=2,label,double fold]B[1]{oo.OtO.oo}
```

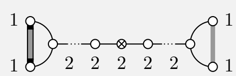


$\mathfrak{sl}(2|2n)^{(2)}$

```
\dynkin[ply=2,label,double edges]D[1]{oo.oto.ooo}
```

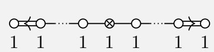



```
\dynkin[ply=2,label,double fold left]D[1]{oo.oto.ooo}
```

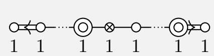


$\mathfrak{osp}(2m|2n)^{(2)}$

```
\dynkin[label,label macro/.code={1}]D[2]{o.oto.oo}
```

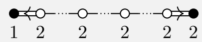



```
\dynkin[label,label macro/.code={1}]D[2]{o.Oto.Oo}
```

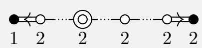


$\mathfrak{osp}(2|2n)^{(2)}$

```
\dynkin[label,label macro/.code=\lablIt{#1},
        affine mark=*]
        D[2]{o.o.o.o*}
```

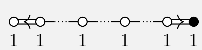



```
\dynkin[label,label macro/.code=\lablIt{#1},
        affine mark=*]
        D[2]{o.O.o.o*}
```

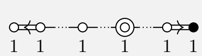


$\mathfrak{sl}(1|2n+1)^4$

```
\dynkin[label,label macro/.code={1}]D[2]{o.o.o.o*}
```

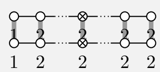



```
\dynkin[label,label macro/.code={1}]D[2]{o.o.O.o*}
```

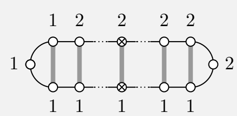


A^1

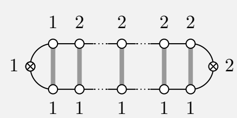
```
\begin{tikzpicture}
    \dynkin[name=upper]A{oo.t.oo}
    \node (Dynkin current) at (upper root 1){};
    \dynkinSouth
    \dynkin[at=(Dynkin current),name=lower]A{oo.t.oo}
    \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,5}{
        \draw[/Dynkin diagram/fold style]
            ($(upper root \i)$) -- ($(lower root \i)$);
    }
    \end{pgfonlayer}
\end{tikzpicture}
```



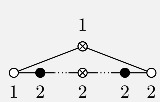

```
\dynkin[fold]A[1]{oo.t.ooooo.t.oo}
```




```
\dynkin[fold,affine mark=t]A[1]{oo.o.ootoo.o.oo}
```

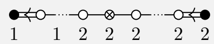



```
\dynkin[affine mark=t]A[1]{o*.t.*o}
```

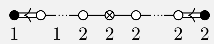


B^1

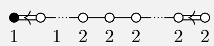
```
\dynkin[affine mark=*]A[2]{o.oto.o*}
```



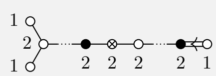

```
\dynkin[affine mark=*]A[2]{o.oto.o*}
```



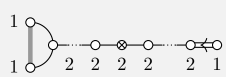

```
\dynkin[affine mark=*]A[2]{o.ooo.oo}
```



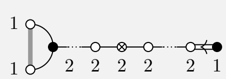

```
\dynkin[odd]A[2]{oo.*to.*o}
```




```
\dynkin[odd,fold]A[2]{oo.oto.oo}
```

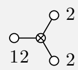



```
\dynkin[odd,fold]A[2]{o*.oto.o*}
```

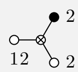


D^1

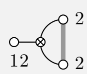
```
\dynkin D{otoo}
```




```
\dynkin D{ot*o}
```

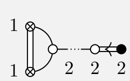



```
\dynkin[fold]D{otoo}
```

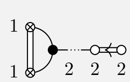


C^1

```
\dynkin[double edges,fold,affine mark=t,odd]A[2]{to.o*}
```

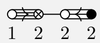



```
\dynkin[double edges,fold,affine mark=t,odd]A[2]{t*.oo}
```

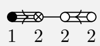


F^1

```
\begin{dynkinDiagram}A{oto*}%
      \dynkinQuadrupleEdge 12%
      \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```

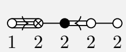



```
\begin{dynkinDiagram}A{*too}%
      \dynkinQuadrupleEdge 12%
      \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```

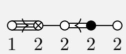


G^1

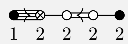
```
\begin{dynkinDiagram}A{ot*oo}%
      \dynkinQuadrupleEdge 12%
      \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



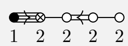

```
\begin{dynkinDiagram}A{oto*o}%
      \dynkinQuadrupleEdge 12%
      \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



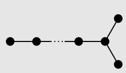
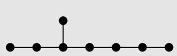
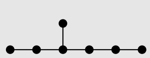
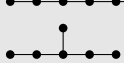
```
\begin{dynkinDiagram}A{*too*}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```




```
\begin{dynkinDiagram}A{*tooo}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



30. Example: the complex simple Lie algebras

$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
A_n		$\frac{1}{n+1}\mathbb{Z}^{n+1}/\left\langle\sum e_j\right\rangle$	$e_i - e_j$	$e_i - e_{i+1}$
B_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
C_n		$\mathbb{Z}^n$	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
D_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$
E_8		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i$ even	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$
E_7		$\frac{1}{2}\mathbb{Z}^8/\langle e_1 - e_2\rangle$	quotient of E_8	quotient of E_8
E_6		$\frac{1}{3}\mathbb{Z}^8/\langle e_1 - e_2, e_2 - e_3\rangle$	quotient of E_8	quotient of E_8
F_4		$\mathbb{Z}^4$	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$

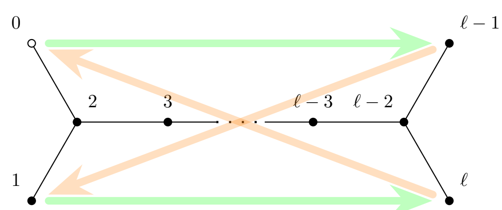
$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
G_2		$\mathbb{Z}^3/\left\langle\sum e_j\right\rangle$	$\pm(1,-1,0)$, $\pm(-1,0,1)$, $\pm(0,-1,1)$, $\pm(2,-1,-1)$, $\pm(1,-2,1)$, $\pm(-1,-1,2)$	$(-1,0,1)$, $(2,-1,-1)$

```
\NewDocumentEnvironment{bunch}{}%
{
   \renewcommand*{\arraystretch}{1}
   \begin{array}{@{}ll@{}}
   \\ \midrule
}{
   \\ \midrule\end{array}
}
\small
\NewDocumentCommand\nct{mm}
{
   \newcolumntype{#1}{>{\columncolor[gray]{.9}}>{$}m{#2cm}<{$}}
}
\nct{G}{.3}
\nct{J}{2.1}
\nct{K}{3}
\nct{R}{3.7}
\nct{S}{3}
\NewDocumentCommand\LieG{}{\mathfrak{g}}
\NewDocumentCommand\W{om}
{
   \ensuremath{
      \mathbb{Z}^{#2}
      \IfValueT{#1}{/\left<#1\right>}
   }
}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GJKRS@{}}
\LieG&
   \text{Diagram}&
   \text{Weights}&
   \text{Roots}&
   \text{Simple roots}\\
\midrule\endfirsthead
\LieG&
   \text{Diagram}&
   \text{Weights}&
   \text{Roots}&
   \text{Simple roots}\\
\midrule\endhead
A_n&
```

```
    \dynkin A{}&
    \frac1{n+1}\W[\sum e_j]{n+1}&
    e_i-e_j&
    e_i-e_{i+1}\\
B_n&
    \dynkin B{}&
    \frac12\W n&
    \pm e_i, \pm e_i \pm e_j, i\ne j&
    e_i-e_{i+1}, e_n\\
C_n&
    \dynkin C{}&
    \W n&
    \pm 2 e_i, \pm e_i \pm e_j, i\ne j&
    e_i-e_{i+1}, 2e_n\\
D_n&
    \dynkin D{}&
    \frac12\W n&
    \pm e_i \pm e_j, i\ne j &
    \begin{bunch}
        e_i-e_{i+1},&i\le n-1\\
        e_{n-1}+e_n
    \end{bunch}\\
E_8&
    \dynkin E8&
    \frac12\W 8&
    \begin{bunch}
        \pm2e_i\pm2e_j,&i\ne j,\\
        \sum_i(-1)^{m_i}e_i,&\sum m_i \text{ even}
    \end{bunch}&
    \begin{bunch}
        2e_1-2e_2,\\
        2e_2-2e_3,\\
        2e_3-2e_4,\\
        2e_4-2e_5,\\
        2e_5-2e_6,\\
        2e_6+2e_7,\\
        -\sum e_j,\\2e_6-2e_7
\end{bunch}\\
E_7&
    \dynkin E7&
    \frac12\W[e_1-e_2]8&
    \quo&
    \quo\\
E_6&
    \dynkin E6&
    \frac13\W[e_1-e_2,e_2-e_3]8&
    \quo&
    \quo\\
F_4&
    \dynkin F4&
    \W4&
    \begin{bunch}
        \pm 2e_i,\\
```

```
        \pm 2e_i \pm 2e_j, \quad i \ne j,\\
        \pm e_1 \pm e_2 \pm e_3 \pm e_4
    \end{bunch}&
    \begin{bunch}
        2e_2-2e_3,\\
        2e_3-2e_4,\\
        2e_4,\\
        e_1-e_2-e_3-e_4
    \end{bunch}\\
G_2&
    \dynkin G2&
    \W[\sum e_j]3&
    \begin{bunch}
        \pm(1,-1,0),\\
        \pm(-1,0,1),\\
        \pm(0,-1,1),\\
        \pm(2,-1,-1),\\
        \pm(1,-2,1),\\
        \pm(-1,-1,2)
    \end{bunch}
&
    \begin{bunch}
        (-1,0,1),\\
        (2,-1,-1)
    \end{bunch}
\end{longtable}
```

31. An example of Mikhail Borovoi




```
\tikzset{
    big arrow/.style={
        -Stealth,
        line cap=round,
        line width=1mm,
        shorten <=1mm,
        shorten >=1mm}
}
\newcommand\catholic[2]{
    \draw[big arrow,green!25!white] (root #1) to (root #2);
}
\newcommand\protestant[2]{
    \begin{scope}[transparency group, opacity=.25]
        \draw[big arrow,orange] (root #1) to (root #2);
    \end{scope}
}
\begin{dynkinDiagram}[%
```

```
    edge length=1.2cm,
    indefinite edge/.style={
        thick,
        loosely dotted
    },
    labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}]
    D[1]{}
    \catholic 06\catholic 17
    \protestant 70\protestant 61
\end{dynkinDiagram}
```

32. Syntax

The syntax is \dynkin[<options>]{<letter>}[<twisted rank>]{<rank>} where <letter> is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, <twisted rank> is 0, 1, 2, 3 (default is 0) representing:

0 finite root system
1 affine extended root system, i.e. of type $^{(1)}$
2 affine twisted root system of type $^{(2)}$
3 affine twisted root system of type $^{(3)}$

and <rank> is

(1) an integer representing the rank or
(2) blank to represent an indefinite rank or
(3) the name of a Satake diagram as in section 5.

The environment syntax is \begin{dynkinDiagram} followed by the same parameters as \dynkin, then various Dynkin diagram and Ti*k*Z commands, and then \end{dynkinDiagram}.

33. Options

*/.style = Ti*k*Z style data,
default : solid,draw=black,fill=black
style for roots like •

o/.style = Ti*k*Z style data,
default : solid,draw=black,fill=white
style for roots like ∘

O/.style = Ti*k*Z style data,
default : solid,draw=black,fill=white
style for roots like ◎

t/.style = Ti*k*Z style data,
default : solid,draw=black,fill=black
style for roots like ⊗

x/.style = Ti*k*Z style data,
default : solid,draw=black,line cap=round
style for roots like ×

X/.style = Ti*k*Z style data,
default : solid,draw=black,thick,line cap=round
style for roots like **×**

continued . . .

Table 24: . . . continued

`affine mark` = o,O,t,x,X,*,
default : `*`
  default root mark for root zero in an affine Dynkin diagram
`arrow shape/.style` = Ti*k*Z style data,
default : `-{Computer Modern Rightarrow[black]}`
  shape of arrow heads for most Dynkin diagrams that have arrows
`arrow style` = Ti*k*Z style data,
default : `black`
  set to override the default style for the arrows in nonsimply laced Dynkin diagrams, including length, width, line width and color
`arrow width` = length,
default : `1.5(root radius)`
  if you change arrow style or shape, use `arrow width` to say how wide your arrows will be
`arrows` = `true` or `false`,
default : `true`
  whether to draw the arrows that arise along the edges
`backwards` = `true` or `false`,
default : `false`
  whether to reverse right to left
`ceref` = `true` or `false`,
default : `false`
  whether to draw roots in a "ceref" style
`Coxeter` = `true` or `false`,
default : `false`
  whether to draw a Coxeter diagram, rather than a Dynkin diagram
`double edges` = Ti*k*Z style data,
default : not set
  set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows)
`double fold` = Ti*k*Z style data,
default : not set
  set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly
`double left` = Ti*k*Z style data,
default : not set
  set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows)
`double fold left` = Ti*k*Z style data,
default : not set

continued . . .

Table 24: ...continued

set to override the fold style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly
double right = TikZ style data,
default : not set
set to override the fold style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows)
double fold right = TikZ style data,
default : not set
set to override the fold style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly
edge label/.style = TikZ style data,
default : text height=0,text depth=0,label distance=-2pt
style of edge labels in the Dynkin diagram, as found, for example, on some Coxeter diagrams
edge length = length,
default : .35cm
distance between nodes in the Dynkin diagram
edge/.style = TikZ style data,
default : solid,draw=black,fill=white,thin
style of edges in the Dynkin diagram
extended = true or false,
default : false
Is this an extended Dynkin diagram?
fold = true or false,
default : true
whether, when drawing Dynkin diagrams, to draw them 2-ply
fold left = true or false,
default : true
whether to fold the roots on the left side of a Dynkin diagram
fold radius = length,
default : .3cm
the radius of circular arcs used in curved edges of folded Dynkin diagrams
fold right = true or false,
default : true
whether to fold the roots on the right side of a Dynkin diagram
fold left style/.style = TikZ style data,
default :
style to override the fold style when folding roots together on the left half of a Dynkin diagram

continued ...

Table 24: ...continued

fold right style/.style = Ti*k*Z style data,
default :
style to override the fold style when folding roots together on the right half of a Dynkin diagram

fold style/.style = Ti*k*Z style data,
default : solid,draw=black!40,fill=none,line width=radius
when drawing folded diagrams, style for the fold indicators

gonality = math,
default : 0
the gonality of a G or I Coxeter diagram

horizontal shift = length,
default : 0
the gonality of a G or I Coxeter diagram

indefinite edge ratio = float,
default : 1.6
ratio of indefinite edge lengths to other edge lengths

indefinite edge/.style = Ti*k*Z style data,
default : solid,draw=black,fill=white,thin,densely dotted
style of the dotted or dashed middle third of each indefinite edge

involution/.style = Ti*k*Z style data,
default : latex-latex,black
style of involution arrows

involutions = semicolon separated list of pairs,
default :
involution double arrows to draw

Kac = true or false,
default : false
whether to draw in the style of [15]

Kac arrows = true or false,
default : false
whether to draw arrows in the style of [15]

label = true or false,
default : false
whether to label the roots according to the current labelling scheme

label* = true or false,
default : false
whether to label the roots at alterative label locations according to the current labelling scheme

label depth = 1-parameter TEX macro,
default : g
the current maximal depth of text labels for the roots, set by giving mathematics text of that depth

label directions = comma separated list,
default :
list of directions to place root labels: above, below, right, left, below right, and so on.

continued ...

Table 24: ...continued

label* directions = comma separated list,
default :
list of directions to place alternate root labels: above, below, right, left, below right, and so on.
label height = ⟨1-parameter TEX macro⟩,
default : b
the current maximal height of text labels for the roots, set by giving mathematics text of that height
label macro = 1-parameter TEX macro,
default : #1
the current labelling scheme for roots
label macro* = ⟨1-parameter TEX macro⟩,
default : #1
the current labelling scheme for alternate roots
make indefinite edge = ⟨edge pair i-j or list of such⟩,
default : {}
edge pair or list of edge pairs to treat as having indefinitely many roots on them
mark = ⟨o,O,t,x,X,*⟩,
default : *
default root mark
name = ⟨string⟩,
default : anonymous
A name for the Dynkin diagram, with anonymous treated as a blank; see section 28
ordering = ⟨Adams, Bourbaki, Carter, Dynkin, Kac⟩,
default : Bourbaki
which ordering of the roots to use in exceptional root systems as in section 19
parabolic = ⟨integer⟩,
default : 0
A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^{i-1} a_i$, $a_i = 0$ or 1, to say that root i is crossed, i.e. a noncompact root
ply = ⟨0,1,2,3,4⟩,
default : 0
how many roots get folded together, at most
reverse arrows = true or false,
default : true
whether to reverse the direction of the arrows that arise along the edges
root radius = ⟨number⟩cm,
default : .05cm
size of the dots and of the crosses in the Dynkin diagram
text style = Ti*k*Z style data,
default : scale=.7

continued ...

Table 24: ...continued

Style for any labels on the roots

`upside down` = `true` or `false`,
default : `false`

whether to reverse up to down

`vertical shift` = ⟨`length`⟩,
default : `.5ex`

amount to shift up the Dynkin diagram, from the origin of Ti*k*Z coordinates.

All other options are passed to TikZ.

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