Residue theorem: Let $f$ be analytic in the region $G$ except for the isolated singularities $a_1, a_2, \ldots, a_m$. If $\gamma$ is a closed rectifiable curve in $G$ which does not pass through any of the points $a_k$ and if $\gamma \approx 0$ in $G$, then

$$\frac{1}{2\pi i} \int_{\gamma} f = \sum_{k=1}^{m} n(\gamma; a_k) \text{Res}(f; a_k).$$

Maximum modulus principle: Let $G$ be a bounded open set in $\mathbb{C}$ and suppose that $f$ is a continuous function on $\overline{G}$ which is analytic in $G$. Then

$$\max \{|f(z)| : z \in \overline{G}\} = \max \{|f(z)| : z \in \partial G\}.$$

Jacobi's identity: Define the theta function $\vartheta$ by

$$\vartheta(t) = \sum_{n=-\infty}^{\infty} \exp(-\pi n^2 t), \quad t > 0.$$

Then

$$\vartheta(t) = t^{-1/2} \vartheta(1/t).$$